Unit 2 Transformations and RIGID MOTIONS

Summary Sheet

1. **Pre-image**: the shape that you start with.
2. **Image**: The new shape after the mapping (transformation) takes place.
3. A **TRANSFORMATION** is when you have the same number of points in the pre-image as in the image.

1 Example of Transformation

![Example of Transformation](image1)

1 NON-Example of a Transformation

![Non-Example of Transformation](image2)

4. **RIGID MOTION** (also known as an isometric transformation) is a transformation that results in an image that is **CONGRUENT** to the pre-image.
   - PRESERVED: Distances, angle measures, parallelism, collinearity.

5. **Orientation** is the direction (clockwise/counter-clockwise) in which the vertices are lettered.
   - **Direct Rigid Motion** is when orientation is preserved from pre-image to image.
   - **Opposite Rigid Motion** is when orientation is reversed from pre-image to image.

**LINE REFLECTIONS REVERSE ORIENTATION!**

6. **TYPES OF ISOMETRIC TRANSFORMATIONS**:
   a. **Line Reflections**: Always Reverses Orientation (AKA Opposite Isometry)
      - $r_{x-axis}(x, y) \rightarrow (x, -y)$
      - $r_{y-axis}(x, y) \rightarrow (-x, y)$
      - $r_{y-x}(x, y) \rightarrow (y, x)$
      - $r_{y-x}(x, y) \rightarrow (-y, -x)$
      - Reflection in any other line, graph and count!
      - Points on line of reflection are IN Variant
      - Pre-image to Image PATHS are: Parallel, Not Equal.

   ![Image is congruent but letters in the opposite direction.](image3)

   ![You can always count the number of spaces (perpendicular to the line of reflection) and then continue that number of spaces to your image point.](image4)

   ![x = # is a VERTICAL LINE](image5)

   ![y = # is a HORIZONTAL LINE](image6)

   ![Congruent image with the same letter order.](image7)

   ![You can always count the number of spaces (to the point of reflection) and then continue that number of spaces to your image point.](image8)

   ![Congruent image](image9)

   ![Point of reflection should be the midpoint between the pre-image and image.](image10)

   ![Points on line of reflection are IN Variant](image11)

   ![Pre-image to Image PATHS are: Parallel, Not Equal.](image12)

   ![Reflection in any other line, graph and count!](image13)

   ![Points on line of reflection are IN Variant](image14)

   ![Pre-image to Image PATHS are: Parallel, Not Equal.](image15)

   ![Reflection in any other line, graph and count!](image16)
c. **Rotations:** Always PRESERVES orientation.  
(AKA Direct Isometry)

Positive rotations Counter Clockwise  
Negative rotations Clockwise  
- $R_{0, 90}(x, y) \rightarrow (-y, x)$  
- $R_{0, 180}(x, y) \rightarrow (-x, -y)$  
- $R_{0, 270}(x, y) \rightarrow (y, -x)$

Points on center of rotation are IN Variant  
Pre-image to Image PATHS are: NOT Parallel, Not Equal.

(+) Rotation  
COUNTER-CLOCKWISE

(-) Rotation  
CLOCKWISE

Equivalent Rotations: example $R_{90}$ is equivalent to $R_{-270}$. The images will be in the same location!

d. **Translations:** Always Direct Isometry.  

- $T_{a, b}(x, y) \rightarrow (x + a, y + b)$  
- $<a, b>$ is vector notation for a translation.  

NO Invariant Points  
Pre-image to Image PATHS are: Parallel, Equal

You can always rotate your paper and note the new coordinates. Then return your paper to the normal position and plot the image point.

You can always count the translation.  
$x + \#$ moves RIGHT  
$x - \#$ moves left  
$y + \#$ moves UP  
$y - \#$ moves DOWN

7. **SYMMETRY:**

a. **Line Symmetry** (reflectional symmetry) A set of points has line symmetry if and only if there is a line, $l$, such that the reflection through $l$ of each point of the set is also a point of the set.

b. **Rotational Symmetry** An object has rotational symmetry if there is a center point around which the object is turned (rotated) a certain number of degrees and the object looks the same.

- **Regular Polygons**  
  
  $\frac{360}{\# \text{ of sides}} = \text{degree of rotational symmetry}$
c. **Point Symmetry**: exists when a figure is built around a single point called the center of the figure. For every point in the figure, there is another point found directly opposite it on the other side of the center.

A simple test to determine whether a figure has point symmetry is to turn it upside-down and see if it looks the same. A figure that has point symmetry is unchanged in appearance by a 180 degree rotation.

![Symmetry Image]

8. **Quadrants**

9. 2 figures are **CONGRUENT** if one figure can be mapped onto the other using one or more rigid motions.
   a. **CLUES**:
      i. **Orientation reversed** look for a single line reflection (or an odd number of line reflections)
      ii. **Paths not parallel** look for a rotation (use patty paper)
      iii. Can always try to translate obvious points 1st then go from there!

10. **Composition of transformations**: when a transformation is performed to get an image point and then that image point is used to find a second image point. **DO not go back to pre-image.**

    ![Composition Diagram]